Int. J. Heat Mass Transfer. Vol. 36, No. 14, pp. 3625–3631, 1993
Printed in Great Britain

0017–9310/93 $6.00+0.00

Conjugate forced convection–conduction analysis of the performance of a cooling and dehumidifying vertical rectangular fin

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(*Received* 4 *March* 1992)

Abstract—A theoretical analysis of the performance of a cooling and dehumidifying vertical rectangular fin has been carried out. Conventional fin theory based on a non-uniform heat transfer coefficient was extended by assuming that the fin is thin and a laminar boundary layer exists around the fin surface. Numerical results for the fin efficiency under various psychrometric and flow conditions were obtained by solving simultaneously the coupled convective boundary layer equation of the fluid and conduction equation of the fin. Under these conditions, numerical results are presented and compared with those of a dry fin. The effect of convection–conduction parameter (CCP) on the fin performance was also investigated.

1. INTRODUCTION

IN CONVENTIONAL air conditioning systems, finned tube heat exchangers are used for air cooling and dehumidifying. Among the factors affecting the thermal performance of such heat exchangers are geometry, materials and psychrometric conditions. When simultaneous heat and mass transfer exist, a considerable change in the coil performance occurs, as a consequence of the change in the fin efficiency. Hence the variation of fin efficiency must be known to determine the overall cooling coil performance. When a phase change occurs, as in dehumidification, the corresponding change in fin efficiency becomes an influential factor in determining coil performance.

Theoretical analysis of the overall performance of cooling fins with dehumidification based on a uniform heat transfer coefficient along the fin has been carried out in the past by many investigators [1–4]. The results show that a significant reduction in fin efficiency occurs as dehumidification is increased. However, analysis based on the assumption of a known and uniform heat transfer coefficient may not be justified as this coefficient varies along the fin due to boundary layer growth. Therefore, a better approach is to take into account the non-uniformity of the heat transfer coefficient by solving the conjugate convective–conductive heat transfer problem. This approach was adopted by Sunden [5], and Sparrow and Chyu [6] for a dry vertical plate fin. They all concluded that the conventional fin theory based on a known and uniform heat transfer coefficient yields a very good prediction for the rate of overall heat transfer of the fin, but cannot predict the local heat fluxes and fin efficiency accurately. Therefore, the case that will be analyzed in this paper is an extension of earlier work in this field.

2. FORMULATION OF THE MODEL

2.1. *Fin energy equation*

When humid air contacts a surface below its dew point temperature, the water vapour condenses on it in filmwise, dropwise or mixed mode, depending on the condition of the surface; a clean surface tends to promote filmwise, and a treated surface dropwise or mixed mode condensation. The presence of condensate on the cooling surface may enhance the heat and mass transfer at the surface due to increased turbulence and effective roughness. The present analysis assumes that at low relative humidities encountered in practice, the condensate thermal resistance to heat flow is negligible because the condensate film is much thinner than the boundary layer in the dehumidification process. This assumption is based on the earlier work investigated in ref. [1]. Under such circumstances, it is expected that the heat transfer coefficient is not significantly influenced by the presence of condensation. However, the effect of mass flux on the temperature profile is accounted for using the Ackermann correction factor [7].

The physical model and coordinate system to which the conservation equations are applied are shown in Fig. 1.

The following conventional simplifying assumptions are made:

1. the heat flow within the fin is one-dimensional;
2. the thermal conductivity of the fin is constant;
3. the temperature at the fin base is uniform and constant;
4. the convective heat transfer can be described using Newton's law of cooling;
5. condensation occurs if the fin surface temperature is below the dew point temperature of the surrounding humid air.

NOMENCLATURE

- b fin width
- C_f vapor enhancement factor
- CCP convection–conduction parameter, $(k_\infty L/k_f t) Re^{1/2}$
- F dimensionless parameter defined by equation (13)
- $h(x)$ local heat transfer coefficient
- k fluid thermal conductivity
- k_f thermal conductivity
- L fin height
- n parameter in equation (23), $(x/T_f - T_\infty)\, d(T_f - T_\infty)/dx$
- Pr Prandtl number, ν/α
- q_l latent heat flux
- q_s sensible heat flux
- q_t total heat flux, $q_s + q_l$
- Q_f fin total heat transfer rate
- Q_t total heat transfer rate at the fin surface
- R ratio of sensible to total heat transfer
- R_{fb} ratio of sensible to total heat transfer calculated at fin base temperature
- Re Reynolds number, $U_\infty L/\nu$
- t fin thickness
- T_f fin surface temperature
- T_{fb} fin base temperature
- T_∞ free stream dry bulb temperature
- U_∞ free stream velocity
- x distance from leading edge
- y distance normal to fin.

Greek symbols
- η similarity variable
- η_{dry} dry fin efficiency
- η_{wet} wet fin efficiency
- Θ_f dimensionless temperature, $(T_\infty - T_f)/(T_\infty - T_{fb})$
- Θ_a Ackermann factor
- ν fluid kinematic viscosity
- ξ dimensionless height, x/L
- ψ stream function.

With these assumptions, the fin energy balance can be written as

$$Q_f(x+\Delta x) = Q_f(x) + Q_t(x) \tag{1}$$

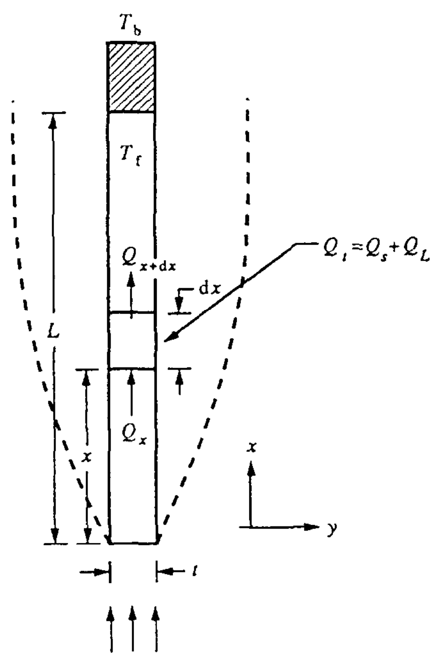

FIG. 1. Problem under consideration.

$$Q_f(x+\Delta x) = Q_f(x) + \frac{dQ_f(x)}{dx}\Delta x. \tag{2}$$

Substituting equation (1) into equation (2) gives

$$\frac{dQ_f(x)}{dx}\Delta x = Q_t(x) \tag{3}$$

where $Q_f(x)$ is the heat conducted within the fin and $Q_t(x)$ is the total heat from the humid air to the fin at x. This consists of both sensible and latent heat transfer. The total heat $Q_t(x)$ can be written in terms of total heat flux $q_t(x)$ as

$$Q_t(x) = 2(b+t)q_t(x)\Delta x. \tag{4}$$

Substituting equation (4) into equation (3) gives

$$\frac{dQ_f}{dx} = 2(b+t)q_t(x). \tag{5}$$

Using the assumption (1) which is conventionally based on the criterion that the fin length be very much larger than the fin thickness, the local conduction within the fin can be obtained from the Fourier law of heat conduction

$$Q_f(x) = -k_f tb\frac{dT_f}{dx}. \tag{6}$$

Substitution of equation (6) into equation (5) gives

$$\frac{d^2T_f}{dx^2} = -\left[\frac{2(b+t)q_t(x)}{k_f tb}\right]. \tag{7}$$

The total amount of heat flux at the fin surface consists of sensible heat flux, $q_s(x)$, due to force convection plus the latent heat flux, $q_l(x)$, of condensation of moisture. Writing this symbolically gives

$$q_t(x) = q_s(x) + q_l(x) = \frac{q_s(x)}{R} \tag{8}$$

where R, the ratio of sensible heat flux to total heat flux, is widely used in psychrometric calculation and its value can lie between zero and unity when simultaneous heat and mass transfer occur. For details of the calculation of R, see ref. [2].

If $h(x)$ is the local heat transfer coefficient at the fin surface, the sensible heat flux due to the cooling of the bulk gas is given by

$$q_s(x) = h(x)(T_\infty - T_f). \tag{9}$$

Equation (9) is similar to that for the convective cooling of a single phase gas. However, the use of this equation to calculate sensible heat flux, when simultaneous heat and mass transfer occur, may not be justified due to the condensation on the fin surface and the effect of mass transfer on the temperature profile.

The presence of condensate, either filmwise or dropwise, can enhance the heat and mass transfer at the interface due to increased turbulence and effective surface roughness. Sardesi *et al.* [8] suggested that this enhancement in heat transfer may be accounted for by introducing in equation (9) an interface enhancement factor C_f. The term C_f has a minimum value of unity for a smooth surface. However, it is likely to be greater than unity at a rough surface, because of increased turbulence. Since it is difficult to theoretically quantify the value of C_f, because of its dependence on geometry and flow conditions, it has to be determined experimentally. Thus to be conservative C_f may be taken as unity.

The effect of mass transfer is to distort the temperature profile in the boundary layer in such a way that the sensible heat transfer is increased. This effect can be accounted for by introducing the Ackermann correction factor, Θ_a having a value of unity when mass flux is negligible and greater than unity when simultaneous heat and mass transfer occur. Taking into account the above corrections, the modified form of equation (9) becomes

$$q_s(x) = C_f \Theta_a h(x)(T_\infty - T_f). \tag{10}$$

In equation (10), as the effect of condensation on boundary layer becomes negligible, C_f and Θ_a tend to unity. Thus in the limiting case of negligible condensation, equation (10) reduces to equation (9), which applies to cooling of a single phase gas. Substituting equations (8) and (10) into equation (7) gives

$$\frac{d^2T_f}{dx^2} = -\left[\frac{2(b+t)C_f\Theta_a h(x)(T_\infty - T_f)}{k_f tbR}\right]. \tag{11}$$

To complete the formulation of the problem, it remains to specify the boundary conditions. At the fin base ($x = L$), the temperature continuity requires that $T_f = T_{fb}$, where T_{fb} is a specified constant. The boundary condition at the fin tip ($x = 0$) is quite complex and cannot be analyzed with a high degree of certainty and is therefore assumed that the fin tip is insulated [5]. The mathematical statement of the boundary conditions is

$$x = 0, \quad \frac{dT_f}{dx} = 0 \tag{12a}$$

$$x = L, \quad T_f = T_{fb}. \tag{12b}$$

To reduce the fin energy equation and the boundary conditions to their essence, the following dimensionless variables are introduced:

$$\Theta_f = \frac{T_\infty - T_f}{T_\infty - T_{fb}}, \quad \xi = \frac{x}{L}, \quad F = \frac{2(b+1)C_f\Theta_a h(x)}{k_f tb}. \tag{13}$$

Introducing these dimensionless variables into equation (11) and rearranging, the following differential equation is obtained:

$$\frac{d^2\Theta_f}{d\xi^2} = \frac{F\Theta_f}{R}. \tag{14}$$

The mathematical statement of the boundary conditions in terms of the transformed variables is

$$\xi = 0, \quad \frac{d\Theta_f}{d\xi} = 0, \quad \text{and} \quad \xi = 1, \quad \Theta_f = 1. \tag{15}$$

Equation (14), when solved with boundary conditions (15), provides a means for obtaining the distribution of the fin temperature Θ_f. The fact that the variation of $h(x)$ is unknown, precludes such a solution at this stage. In order to obtain an equation that governs the variation of sensible heat transfer coefficient, h, with x, consideration has to be given to the boundary layer equations.

2.2. *Boundary layer equations*

The variation of sensible heat transfer coefficient $h(x)$ along the fin may be obtained by solving the appropriate form of the boundary layer equations. Assuming steady, laminar, incompressible flow and negligible viscous dissipation and recognizing that $dp/dx = 0$, the boundary layer equations can be expressed as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{16}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \tag{17}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\nu}{Pr}\frac{\partial^2 T}{\partial y^2}. \tag{18}$$

The appropriate boundary conditions are

$$u(x,0) = v(x,0) = 0 \quad \text{and} \quad u(x,\infty) = U_\infty \tag{19a}$$

$$T(x,0) = T_f, \quad T(x,\infty) = T_\infty. \tag{19b}$$

Equations (16)–(18) and boundary conditions (19a) and (19b) do not admit similarity solution. The non-similarity arises from the non-uniform temperature distribution of the fin. The above equations and the boundary conditions are transformed into dimensionless forms by introducing the Falkner–Skan transformation as follows:

$$f(\eta) = \frac{\psi}{U_\infty \sqrt{(\nu x/U_\infty)}}, \quad \text{and} \quad \eta = y\sqrt{\left(\frac{U_\infty}{\nu x}\right)} \tag{20}$$

along with the dimensionless temperature $g(x,\eta)$ defined by

$$g(x,\eta) = \frac{T_f(x) - T(x,\eta)}{T_f(x) - T_\infty}. \tag{21}$$

Substituting these expressions into equations (16)–(18) we then obtain

$$f''' + \frac{1}{2}ff'' = x\left(f'\frac{\partial f'}{\partial x} - f''\frac{\partial f}{\partial x}\right) \tag{22}$$

$$\frac{1}{Pr}g'' + \frac{1}{2}fg' + n(1-g)f' = x\left(f'\frac{\partial g}{\partial x} - g'\frac{\partial f}{\partial x}\right). \tag{23}$$

The appropriate boundary conditions in terms of the above variables become

$$f(x,0) = f'(x,0) = 0, \quad f'(x,\infty) = 1,$$
$$g(x,0) = 0, \quad g(x,\infty) = 1. \tag{24}$$

In the foregoing equations, the primes stand for partial derivatives with respect to η. When equations (22) and (23) are solved numerically, the convective coefficient, $h(x)$, expressed in terms of the transformed variables, can be calculated using the following equation:

$$h(x) = k\left(\frac{U_\infty}{\nu x}\right)^{1/2}\left.\frac{\partial g}{\partial \eta}\right|_{\eta=0}. \tag{25}$$

3. NUMERICAL SOLUTION

The computer program developed to solve the governing differential equations (22) and (23) was based on an accurate implicit finite-difference technique known as the Keller Box method [9]. This technique has several desirable features that makes it appropriate for the solution of boundary layer equations. The equations are first converted into a system of first-order differential equations which are then expressed in finite-difference form. The resulting non-linear finite difference equations are then solved by Newton's iterative method. The numerical solutions were obtained by a marching process in the stream-wise direction, starting at the leading edge of the fin.

The fin energy equation (14) was solved by an iterative method using the successive over relaxation (SOR) technique. The solution was started by solving the boundary layer equations with initial fin temperature distribution obtained from the conventional fin theory. The heat transfer coefficient calculated from equation (25) was then used as input to the fin energy equation. This procedure of solving the boundary layer equations and fin energy equation was continued until convergence was achieved.

The computer program was run on an IBM 4341 computer with double precision arithmetic and the accuracy of the numerical solution procedure was established by comparing the dry fin results with those of ref. [5].

4. RESULTS AND DISCUSSION

4.1. *Temperature distribution*

Shown in Fig. 2 are the numerical results for the dimensionless temperature distribution, Θ_f, plotted against the non-dimensional fin height, ξ, for a dry fin and for various values of the CCP parameter. It can be seen that, with increasing CCP value, the variation in fin surface temperature along the fin becomes greater. Also, it is evident from this figure that the conventional fin theory based on a known and uniform heat transfer coefficient predicts a lower base to tip temperature variation than actually prevails using the conjugate theory. These results are also in agreement with those obtained in ref. [5].

Shown in Fig. 3 is the dimensionless temperature distribution for a wet fin (with dehumidification) for different values of CCP. It shows that the conventional fin theory predicts lower fin surface temperature than the conjugate theory. The difference is similar to those of a dry fin.

The effect of various free stream temperatures for a given relative humidity and fin base temperature on fin temperature distribution is shown in Fig. 4. It shows clearly that the temperature profiles of a wet surface fin lie below those of a dry surface fin. As the ambient air temperature increases, the departure of the temperature profiles from the dry surface curves becomes greater. This is attributed to the increase

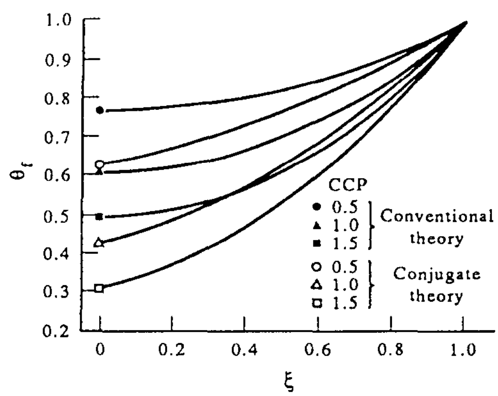

FIG. 2. Effect of CCP parameter on dry fin temperature distribution.

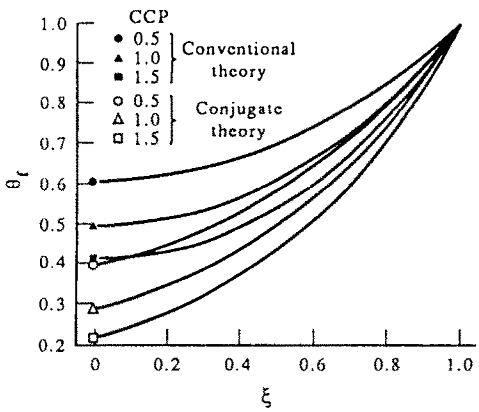

FIG. 3. Effect of CCP parameter on wet fin temperature distribution.

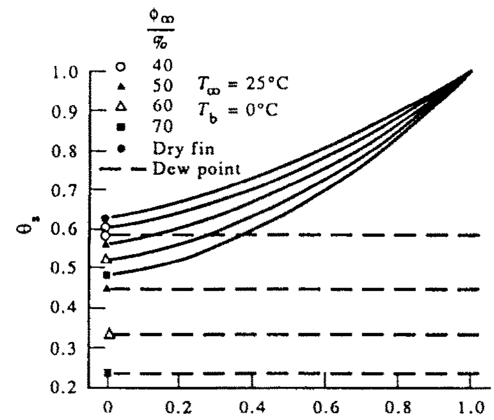

FIG. 5. Effect of relative humidity on fin temperature distribution.

of temperature and the moisture content differences between the bulk and the phase interface, which consequently leads to higher sensible and latent heat transfer, other conditions being kept constant.

The effect of relative humidity on temperature profiles, Fig. 5, was also investigated. It was found that the increase in relative humidity resulted in a higher fin surface temperature due to the increase of the motive force of water vapor molecules condensing on the fin surface. This resulted in a higher latent heat release at the fin surface and a higher fin surface temperature. The increase in fin base temperature decreased the driving potential for both heat and mass transfer, and hence decreased the fin surface temperature.

4.2. *Comparison of dry and wet fin heat transfer*

Computations of the fin heat transfer with and without dehumidification were performed for different values of ambient air temperature, other psychrometric conditions being kept constant. For a given condition, Fig. 6 shows that the local heat flux decreases rapidly with an increase in distance from the fin leading edge and then starts to increase. This can be attributed to the growth of the boundary layer at the leading edge with an infinite value of heat transfer coefficient and also decrease in the fin surface temperature towards the fin root. Also, it is evident that the local heat transfer for a dry fin is below that of a wet fin. The difference between the dry and wet fin becomes greater as the air temperature increases. The increase in relative humidity, Fig. 7, increased the moisture content difference between the bulk and the fin surface, which consequently leads to higher heat transfer.

The variation of total heat transfer with CCP parameter for various ambient air temperature is shown in Fig. 8. It is evident that as CCP or ambient air

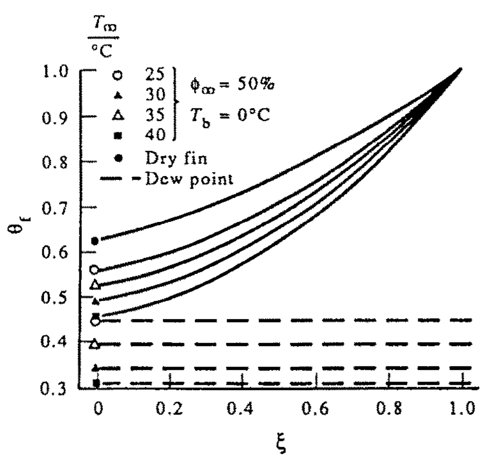

FIG. 4. Effect of ambient air temperature of fin temperature distribution.

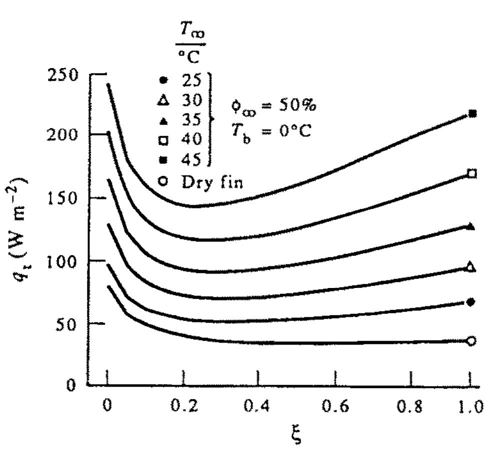

FIG. 6. The variation of heat flux along the fin with air temperature.

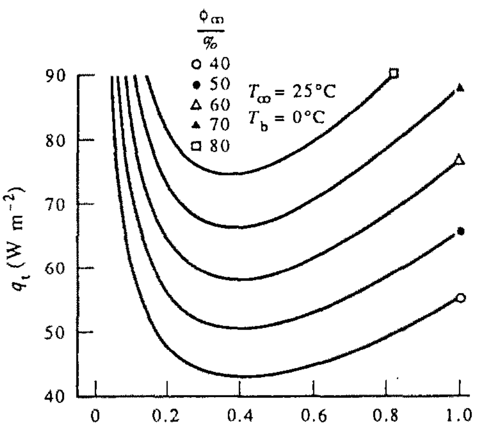

FIG. 7. The variation of heat flux along the fin with relative humidity.

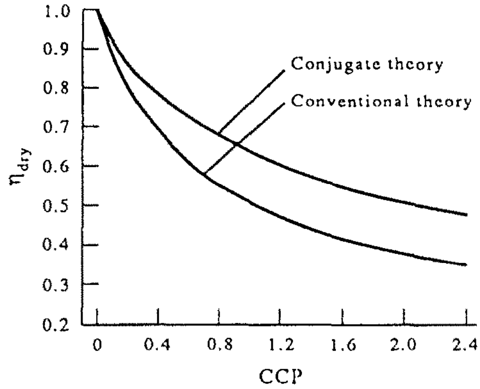

FIG. 9. The variation of dry fin efficiency with CCP parameter.

temperature increases, heat transfer to the fin increases due to the increase in driving potential, since larger CCP values represent a fin with a lower thermal conductivity and higher surface temperature in the neighborhood of the fin root.

4.3. *Fin efficiency*

The loss in performance of the real fin is characterized as the fin efficiency. This is defined as the ratio of actual heat transfer through a fin surface to heat transfer of an ideal fin at the base temperature and may be written in dimensionless form as

$$\eta_{\mathrm{dry}} = \int_0^1 \Theta_{\mathrm{f}}\,\mathrm{d}\xi. \tag{26}$$

The wet fin efficiency may be defined as the ratio of heat actually transferred if the entire fin were at the temperature of the fin base. The simplified expression for fin efficiency with dehumidification in terms of non-dimensional variables becomes

$$\eta_{\mathrm{wet}} = \int_0^1 \left(\frac{R_{\mathrm{fb}}}{R}\right)\Theta_{\mathrm{f}}\,\mathrm{d}\xi. \tag{27}$$

The efficiency of a dry fin as a function of the CCP parameter is shown in Fig. 9. In this figure the results of conventional theory are also depicted. The difference in efficiency for low values of the CCP parameter is less than 5%. Therefore, the simple conventional fin theory might be sufficient for engineering calculation.

Shown in Fig. 10 are the computed dry and wet fin efficiencies plotted as a function of the CCP parameter for various dry bulb temperatures and given psychrometric conditions. The definition of efficiency of a wet surface fin is similar to that for a dry surface fin. However, the departure of wet fin efficiency from the dry surface condition is due to the additional latent heat released during dehumidification process.

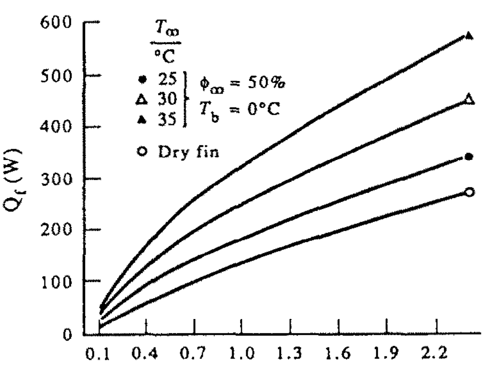

FIG. 8. The variation of total heat transfer with CCP parameter and air temperature.

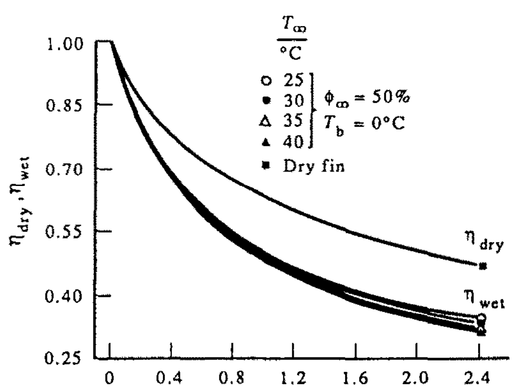

FIG. 10. Comparison of dry and wet fin efficiency with CCP parameter.

5. CONCLUSIONS

A numerical study has been conducted to analyze the conjugate convection and conduction heat transfer characteristics of humid air flow over a vertical rectangular fin. The effect of various parameters and psychrometric conditions on its heat transfer characteristic has been investigated. In general it has been found that the local heat flux, the fin temperature and fin efficiency were affected by non-uniformity of heat transfer coefficient, the CCP parameter and psychrometric conditions. A significant reduction in fin efficiency occurs when the amount of dehumidification is increased. The reduction in wet fin efficiency from that with dry surface conditions depends on the difference in temperature and relative humidity between the surrounding humid air and the fin.

Acknowledgements—This research is supported by the University of Shiraz (grant No. 69-EN-584-311) whose assistance is hereby gratefully acknowledged.

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